Table 2 Distribution identification for extreme value distributions

Component life	Minimum extreme (4-out-of-4 case)	Maximum extreme (1-out-of-4 case)
Exponential	Gamma (E)[a]	Gamma (E)
Location parameter	0.02	375.86
Scale parameter	731.36	2135.82
Shape parameter	1.01	2.72
Uniform	Weibull (E)	Extreme value type A
Location parameter	2500.03	3370.41
Scale parameter	206.28	120.79
Shape parameter	1.13	——
Normal	Weibull (E)	Log normal
Location parameter	982.34	0.0
Scale parameter	1631.93	8.16
Shape parameter	4.77	0.10

[a]E is extended version of the distribution specified.

obtain the PDF of the final random variable of interest. For example, if each engine has a normally distributed life, the maximum life in this scenario has a PDF that fits to a log-normal distribution with location, scale, and shape parameter values of 0, 8.16, and 0.10. Let us assume that the trip requires 3800 time units. What is the probability that the mission will be accomplished as one or more engines fail as time goes on? The reliability equation (26) is used to find $R(3800) = 0.2018$. For 2500 time units, the same reliability is 0.9996. Simulation runs[9] have ranged from a minimum of 2131 to a maximum of 5108 time units of duration.

These probabilities are exact answers. If the voyage requires 3800 time units of flight, then there is only 0.2018 probability of reaching the destination even though only one of the four engines is sufficient to power the spacecraft. Decision makers can use this information in determining whether and how to enhance the performance of the engines. If the analytical formula is not available, simulation results can be used. Log-normal CDF formula value or the probability of failure by time 3800, using the parameters shown in Table 2 for this case, is 0.794. Then, reliability is 0.2060 and is higher than the exact value of 0.2018. The very small difference between these two values is similar to the closeness between the simulated and analytical parameters shown in Table 1. If plotted, this case looks near normal as the shape parameter of log-normal is close to zero (0.10) suggesting this normality. In Table 2, the mean and median are also close. It is possible to use normal distribution with a parameter set of 3510.53 and 354.04 (simulation results) without having to identify the distribution itself. Then, $R(3800)$ is 0.2080 and is slightly more inaccurate than the one found using log-normal CDF.

For the 4-out-of-4 case, the reliability for 3800 time units or the probability that the flight can last at least 3800 time units is zero. There is no statistical chance for this mission to succeed although simulation results have ranged from 738 to 3807 time units.[9] In 1 (possibly 2 or 3) of the 26,000 replications the flight duration exceeded 3,800 time units. Statistically, the observation of 3807 and the possible few more between 3800 and 3807 have no significance. It is still true that the mission may have 1 or 2 chances out of 26,000 to succeed. The reliability drops to 0.5524 for 2500 time units compared to 0.9996 for the 1-out-of-4 case as would be expected if all four engines are needed. Table 2 shows that the simulated PDF of this 4-out-of-4 case is distributed according to a Weibull distribution with a parameter set of 982.34, 1631, and 4.77. The Weibull CDF for 2500 time units is 0.5070. Then, the reliability is 0.4930 and is considerably off from the exact value of 0.5524. If Weibull CDF is not used and a normal approximation is made, the reliability value is 0.479 and is more inaccurate.

Conclusion

To maintain the continuing pace of pioneering planetary missions, spacecraft must be highly reliable. This study has provided some simple probability and simulation-based results and concepts that can be used in this effort. Analytical expressions presented are simple, but none has been found readily available in relevant literature following an intense search. Rare texts have complicated mathematical developments, which are often intractable, and it is hard to find the needed information.

The closeness of the simulated and analytical parameters in Table 1 is encouraging for the value of simulation, which is simply an experiment. In one example given earlier, simulation did not perform well. That discrepancy can be partly attributed to inherent statistical errors in any goodness-of-fit tests as well as the simulation process itself.

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Maximum-Payload Transfers to Geosynchronous Orbit Using Arcjet Thrusters

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Introduction

THE payload benefits associated with the use of electric propulsion (EP) for performing near-Earth-orbit transfers has been investigated by several authors.[1–3] However, the utilization of a low-thrust engine for transferring a payload from low Earth orbit (LEO) to geosynchronous orbit (GEO) is a far-term application given the current level of technology of electric propulsion.[4] Current operational uses of EP include on-orbit maneuvers such as stationkeeping and drag make-up.[4] A potential current or near-term application of electric propulsion involves GEO orbit circularization in the case of the chemical apogee engine failure. Geosynchronous spacecraft are usually injected into an elliptical geosynchronous transfer orbit (GTO) with an apogee at GEO altitude and a perigee at LEO altitude. In 1989, the GSTAR-3 satellite utilized the hydrazine resistojet engine designed for on-orbit stationkeeping to circularize the GTO after the apogee engine failed.[4]

In this Note, the use of a combined chemical–electric propulsion system for a LEO–GEO transfer is investigated. The proposed mission scenario involves a chemical insertion into GTO, followed

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by a chemical apogee burn to partially raise perigee and reduce inclination, and finally a low-thrust orbit transfer to equatorial GEO using arcjet thrusters. The objective is to obtain the optimal propulsion modes that maximize the spacecraft's payload in GEO and to compare this optimal combined chemical–EP strategy with an all-chemical-propulsion LEO–GEO transfer.

Mission and Spacecraft Definition

LEO–GEO Mission

For this study, the spacecraft is assumed to be injected into GTO by an Atlas IIAS launch vehicle. The resulting elliptical transfer orbit has a perigee altitude of 167 km, an apogee altitude of 35,786 km (GEO), and an inclination of 26.5 deg. The total spacecraft mass in GTO after the perigee burn is 3833 kg. These parameters represent a typical GTO as presented in Ref. 5. The apogee chemical thruster is a bipropellant system with a specific impulse (I_{sp}) of 314 s. Arcjet thrusters are used for the EP system since they have been identified as a good candidate for the dual role of stationkeeping and primary propulsion.[6] The arcjet propulsion system has a constant input power of 7.5 kW, an I_{sp} of 600 s, and an engine efficiency of 32%. Therefore, the resulting constant thrust magnitude is 0.815 N, and the constant propellant mass flow rate is 11.97 kg/day. The arcjet engines are also to be utilized for GEO stationkeeping.

Spacecraft Mass Analysis

The total mass m_{GTO} of a spacecraft with chemical and electric stages after insertion into GTO is

$$m_{\mathrm{GTO}} = m_{\mathrm{p}_c} + m_{\mathrm{dry}_c} + m_{\mathrm{p}_e} + m_{\mathrm{tank}_e} + m_{\mathrm{pp}} + m_{\mathrm{net}} \qquad (1)$$

The subscripts c and e represent the chemical and electric propulsion stages, respectively. The variable m_{p} is the propellant mass, m_{dry_c} is the dry mass of the chemical stage, m_{tank_e} is the tank mass of the electric stage, m_{pp} is the power- and propulsion-system mass of the electric stage, and m_{net} is the net mass. The spacecraft's net mass represents the usable mass for payload plus the basic spacecraft structural mass. The injected mass $m_{\mathrm{GTO}} = 3833$ kg represents the launch capability of the Atlas IIAS vehicle. The propellant masses m_{p_c} and m_{p_e} are calculated from the rocket equation

$$m_{\mathrm{p}} = m_{\mathrm{i}}(1 - e^{-\Delta V/g I_{\mathrm{sp}}}) \qquad (2)$$

where m_{i} is the initial mass prior to a propulsive maneuver, ΔV is the velocity change, g is the gravitational acceleration at sea level, and I_{sp} is the specific impulse of the propulsion stage. The dry mass m_{dry_c} includes the structural, engine, and tank mass of the chemical stage and is assumed to be 12% of the chemical propellant mass.[7] The tank mass m_{tank_e} is 8% of the total propellant for the electric stage.[4] The power- and propulsion-system mass of the electric stage, m_{pp}, is the product of electric input power P and specific mass α. The specific mass for a power and propulsion system composed of gallium arsenide solar array cells and arcjet thrusters is fixed at 30 kg/kW, which results in $m_{\mathrm{pp}} = 225$ kg.

Mission Analysis

The net mass m_{net} can be expressed using Eq. (1) and the previous mass definitions as

$$m_{\mathrm{net}} = 3833 - 1.12 m_{\mathrm{p}_c} - 1.08 m_{\mathrm{p}_e} - 225 \qquad (3)$$

Therefore, to maximize m_{net}, the optimal combination of propellant for the chemical and electric stages must be determined. In other words, we must find what portion of the three-dimensional GTO–GEO transfer is performed initially by a chemical GTO apogee burn and how the remaining portion of the transfer is completed by the arcjet thrusters so that m_{net} is maximized.

This problem is solved by parametrically varying the magnitude and orientation of the chemical GTO apogee burns in order to produce a range of intermediate inclined elliptical orbits. These intermediate orbits provide initial conditions for the subsequent low-thrust transfers to equatorial GEO. Table 1 presents a range of chemical GTO apogee burns designed to raise the perigee and reduce the inclination. Four discrete perigee raises Δr_p and four discrete inclination reductions Δi are produced by the respective velocity changes ΔV and out-of-plane thrust pointing angles ψ as indicated in Table 1. The impulsive chemical apogee burn is assumed to have components along the apogee velocity vector and normal to the original GTO plane. The pointing angle ψ is measured from the GTO plane to the impulsive thrust vector. Table 1 also presents the propellant mass m_{p_c} and the orbital elements (semimajor axis a, eccentricity e, and inclination i) of the intermediate orbits as a result of the apogee burn. Case 1 corresponds to the absence of an apogee burn, and case 16 corresponds to a complete GTO–GEO transfer via the chemical apogee burn.

Next, the optimal minimum-fuel low-thrust transfers from the initial conditions presented in Table 1 to equatorial GEO are obtained. These minimum-fuel transfer problems are solved by the low-thrust trajectory optimization program SECKSPOT,[8] which utilizes an indirect optimization method and solves the two-point boundary-value problem via a multiple shooting method. SECKSPOT uses orbital averaging for the governing equations of motion and simulates Earth-shadow effects, Earth oblateness, and solar-cell degradation due to the radiation belts. Once the minimum-fuel transfer to GEO is computed, the additional propellant required by the electric stage for stationkeeping is calculated using Eq. (2). A total annual ΔV budget of 50 m/s for east–west and north–south stationkeeping over a spacecraft lifetime of 10 years is assumed.[4]

Results

The resulting optimal payload fractions $m_{\mathrm{net}}/m_{\mathrm{GTO}}$ for the minimum-fuel transfers from the 16 initial conditions outlined in Table 1 are depicted in Fig. 1. Clearly, the propulsion strategy for maximum net mass in GEO is the all-EP GTO–GEO transfer without the use of a chemical apogee rocket. The resulting maximum payload fraction is 0.498 for the all-EP transfer, compared to 0.397 for the all-chemical-propulsion transfer. Therefore, the all-electric GTO–GEO transfer yields a 25.4% increase in payload over the all-chemical transfer. Figure 1 also indicates that if the entire plane change is performed by a chemical apogee burn, then the payload fraction is essentially the same for the variety of combined chemical–electric-propulsion maneuvers for perigee raise.

The resulting low-thrust orbit transfer time, subsequent power degradation, and final spacecraft mass and net mass in GEO for each of the 16 cases detailed in Table 1 are presented in Table 2. The low-thrust transfer time is highest at 121 days for the all-EP transfer and is lowest at 14.8 days for case 12. Solar-cell degradation due to time spent traversing the radiation belt is calculated by SECKSPOT as a percentage loss of the available power at the beginning of the mission. Power degradation is worst at about 5.5% for cases 1–4 which do not employ a perigee raise via the chemical apogee burn. Power degradation decreases as perigee is raised by the apogee burn but remains less than 1% for cases 5–16. The total transfer time and power loss are significantly less than for the corresponding all-EP orbit transfer from LEO to GEO.

Table 1 Chemical apogee burns and resulting intermediate orbits

Case	ΔV, m/s	ψ, deg	m_{p_c}, kg	Δr_p, km	Δi, deg	a, km	e	i, deg
1	0	——	0.0	0	0.0	24,355	0.731	26.5
2	245.4	94.42	293.6	0	8.83	24,355	0.731	17.67
3	489.6	98.84	563.5	0	17.67	24,355	0.731	8.83
4	730.7	103.25	809.6	0	26.5	24,355	0.731	0.0
5	803.6	0.0	880.4	11,873	0.0	30,291	0.392	26.5
6	858.1	25.40	932.2	11,873	8.83	30,291	0.392	17.67
7	1003.2	46.50	1065.7	11,873	17.67	30,291	0.392	8.83
8	1203.7	62.72	1240.1	11,873	26.5	30,291	0.392	0.0
9	1217.6	0.0	1251.8	23,746	0.0	36,228	0.164	26.5
10	1260.4	20.02	1287.5	23,746	8.83	36,228	0.164	17.67
11	1380.3	38.19	1384.7	23,746	17.67	36,228	0.164	8.83
12	1557.0	53.68	1521.2	23,746	26.5	36,228	0.164	0.0
13	1480.7	0.0	1463.3	35,619	0.0	42,164	0.0	26.5
14	1519.5	18.10	1492.9	35,619	8.83	42,164	0.0	17.67
15	1629.4	34.94	1575.0	35,619	17.67	42,164	0.0	8.83
16	1795.1	49.84	1693.3	35,619	26.5	42,164	0.0	0.0

Table 2 GTO–GEO transfers

Case	EP transfer time, days	Power degradation, %	m_{GEO}, kg	m_{net}, kg
1	121.0	5.22	2,458.9	1,907.7
2	104.4	5.56	2,360.0	1,797.8
3	91.3	5.57	2,236.3	1,664.3
4	82.6	5.57	2,086.4	1,505.8
5	79.0	0.77	2,013.4	1,430.5
6	62.3	0.74	2,157.7	1,571.6
7	45.9	0.63	2,219.6	1,627.7
8	37.9	0.53	2,140.2	1,541.9
9	67.5	0.40	1,774.2	1,178.4
10	49.0	0.42	1,960.0	1,361.3
11	27.7	0.33	2,117.1	1,513.2
12	14.8	0.19	2,134.2	1,524.7
13	60.8	0.11	1,641.6	1,038.4
14	43.1	0.15	1,824.2	1,218.3
15	22.3	0.13	1,990.8	1,380.3
16	——	0.00	2,139.7	1,523.3

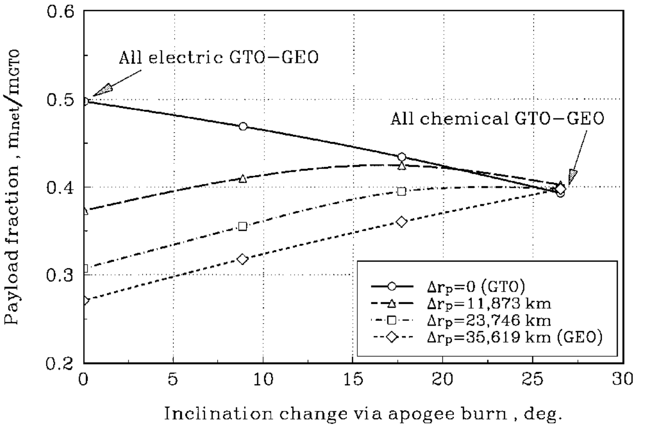

Fig. 1 Maximum payload fraction for GTO–GEO transfer.

Conclusions

A maximum-payload problem for a LEO–GEO transfer using a combined chemical–electric propulsion system with arcjet thrusters has been formulated and solved. The problem is solved by parametrically varying the magnitude and direction of the chemical GTO apogee burn and solving for the subsequent EP transfers with a low-thrust trajectory optimization code. The payload is maximized when the entire GTO–GEO transfer is performed by the electric propulsion stage, and the optimal transfer requires 121 days with a 5.2% loss in solar power. This optimal strategy for the LEO–GEO mission can provide an additional 25.4% payload capability compared to the corresponding all-chemical LEO–GEO mission.

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